William I. Fine Theoretical Physics Institute
University of Minnesota

A possible gateway to η_b: $\chi_{b0}(2P) \to \eta \, \eta_b$

M.B. Voloshin

William I. Fine Theoretical Physics Institute, University of Minnesota,

Minneapolis, MN 55455

and

Institute of Theoretical and Experimental Physics, Moscow, 117259

Abstract

It is argued that the branching ratio for the decay $\chi_{b0}(2P) \to \eta \, \eta_b$ can reach few permil as a result of the enhancement of the η emission by the axial anomaly in QCD. This might make the discussed process practical for an experimental search for the η_b.

The ground 1^1S_0 state of bottomonium, the pseudoscalar η_b resonance, is expected to have mass approximately 40 ± 10 MeV below that of the Υ resonance. An early QCD calculation[1] has estimated the hyperfine splitting at about 35 MeV, while the latest perturbative QCD calculation[2] in the next-to-next-to-leading order provides the central estimated value of the splitting as 39 MeV with a theoretical uncertainty of about 10 MeV (and an additional uncertainty due to the current knowledge of the QCD coupling α_s). The preliminary lattice calculations[3] favor a somewhat larger value of the splitting, up to approximately 50 MeV. Thus an experimental observation of the η_b resonance, besides being of a general interest for the study of the heavy quarkonium, would provide an important input into the current theoretical approaches.

It is however known that experimentally the η_b is quite elusive: the rate of the allowed M1 radiative transition $\Upsilon \to \gamma\,\eta_b$ is greatly suppressed by the small value of the hyperfine splitting, while analogous transitions from the excited $^3S_1$ bottomonium resonances, $\Upsilon(2S) \to \gamma\,\eta_b$ and $\Upsilon(3S) \to \gamma\,\eta_b$, are forbidden in the nonrelativistic limit by the vanishing overlap of the wave functions. Thus the latter transitions proceed only due to the relativistic effects, which are quite small in bottomonium and are highly model-dependent. The purpose of the present letter is to point out that the decay chain $\Upsilon(3S) \to \gamma\,\chi_{b0}(2P)$ followed by $\chi_{b0}(2P) \to \eta\,\eta_b$ can provide an alternative realistic approach to an experimental search for the η_b. The first transition in this chain is well known[1] and is a reliable source of the $\chi_{b0}(2P)$ resonance. (The CLEO data sample contains[5] $(225 \pm 7) \times 10^3$ observed events with the decay $\Upsilon(3S) \to \gamma\,\chi_{b0}(2P)$.) It will be argued here that the branching ratio for the decay $\chi_{b0}(2P) \to \eta\,\eta_b$, although still uncertain, may be as large as few permil, due to a considerable enhancement of the η emission in the quarkonium transition by the axial anomaly in QCD. Depending on the experimental technique, the background conditions for the discussed here decay chain may be more favorable than in a search for the η_b by the direct radiative transition from $\Upsilon(3S)$.

The amplitude of the discussed here η transition in bottomonium carries the usual suppression corresponding to breaking of the flavor SU(3) as well as a suppression by the factor m_b^{-1} corresponding to the spin-flip of the heavy b quark. However these factors are to some extent compensated by that the considered decay is an S wave process and by the enhance-

---

[1]The PDG[4] value for the branching ratio $B(\Upsilon(3S) \to \gamma\,\chi_{b0}(2P))$ is $5.4 \pm 0.6\%$, while the latest CLEO measurement[5] yields $6.77 \pm 0.20 \pm 0.65\%$.

ment of the η production by soft gluonic field, expressed by the relation[6]

$$\langle\eta|G^a\tilde{G}^a|0\rangle = 8\pi^2 \sqrt{\frac{2}{3}} f_\eta m_\eta^2 \,, \tag{1}$$

which is a consequence of the anomaly in the SU(3) flavor singlet axial current in QCD[2]. The constant f_η is the annihilation constant for the η meson, analogous to the pion constant $f_\pi \approx 130$ MeV. One can also notice that eq.(1) contains the flavor SU(3) breaking factor $m_\eta^2 \propto m_s$.

The relation of the amplitudes of hadronic transitions in heavy quarkonium to the matrix elements of the type shown in eq.(1) arises within the description[7] of the hadronic transitions in terms of the multipole expansion in QCD[7, 8]. Within this approach the heavy quarkonium is considered as a compact object whose interaction with the soft gluonic fields can be expanded in multipoles, while the production of the light hadrons in the transition is described[9, 10] by matrix elements analogous to that in eq.(1). However there is a serious uncertainty within this approach associated with evaluation of the heavy quarkonium transition amplitudes resulting from the interactions with gluonic field. This is especially true for transitions from the states whose radial wave function has nodes, such as the discussed here transition $2P \rightarrow 1S$, where considerable cancellations in the overlap integral do take place. In lieu of a better model-independent approach, another known $2P \rightarrow 1S$ transition, $\chi_{b0}(2P) \rightarrow \gamma\,\Upsilon$, is used here for an estimate of the scale of the corresponding matrix element.

The decay $\chi_{b0}(2P) \rightarrow \eta\,\eta_b$ is induced by the interference of the E1 and M1 terms in the multipole expansion in QCD. The Hamiltonian describing these two terms can be written as

$$H_{E1} + H_{M1} = -\frac{1}{2}\xi^a\,\vec{r}\cdot\vec{E}^a(0) - \frac{1}{2\,m_b}\,\xi^a\,(\vec{s}_1 - \vec{s}_2)\cdot\vec{B}^a(0) \,, \tag{2}$$

where $\xi^a = t_1^a - t_2^a$ is the difference of the color generators acting on the quark and antiquark, $\vec{s}_1 - \vec{s}_2$ is the difference of the corresponding spin operators, and $\vec{r}$ is the vector for relative position of the quark and the antiquark. Finally, $\vec{E}^a$ and $\vec{B}^a$ are the chromoelectric and the chromomagnetic components of the gluon field strength.

In a nonrelativistic quarkonium the spin, orbital, and the radial degrees of freedom factorize. Thus using the Hamiltonian in eq.(2) in the second order and retaining only the relevant interference term, one can readily find the amplitude of the discussed transition in

---

[2]The normalization of the gluonic field used here includes the QCD coupling g, which normalization corresponds to writing the gluon Lagrangian as $L_g = -G^2/(4g^2)$.

the form

$$A(\chi_{b0}(2P) \to \eta \, \eta_b) = \frac{1}{92 \, m_b} \langle \eta | \vec{E}^a \cdot \vec{B}^a | 0 \rangle \, J = \frac{\pi^2}{48} \sqrt{\frac{2}{3}} \frac{f_\eta \, m_\eta^2}{m_b} \, J \,, \tag{3}$$

where the axial anomaly relation (1) is taken into account, and J is the radial part of the quarkonium transition amplitude, depending on the overlap of the radial wave functions R_{2P} and R_{1S} as

$$J = \langle R_{1S} | \, r \, \xi^c \, \mathcal{G}_P \, \xi^c \, | R_{2P} \rangle + \langle R_{1S} | \, \xi^c \, \mathcal{G}_S \, \xi^c \, r \, | R_{2P} \rangle \,. \tag{4}$$

Here $\mathcal{G}$ stands for the Green function of the heavy quarkonium in the color octet state. Clearly the first term contains its P wave part, while the second term contains the S wave part of $\mathcal{G}$. At present it is still not clear how to calculate this Green function, which is the main source of uncertainty in estimating the absolute rates of the hadronic transitions. It is believed to be mainly contributed by the states above the open flavor threshold[8]. In this situation for an estimate one can replace this Green function by a local operator $\mathcal{G} \to 1/\Delta$ with Δ being the "effective energy gap" to the states contributing to $\mathcal{G}$. Such approximation is in a reasonable agreement with the data on the transitions $\psi(2S) \to \pi\pi \, J/\psi$ and $\Upsilon(2S) \to \pi\pi \, \Upsilon$, with $\Delta \sim 1$ GeV. Adopting this approximation and taking into account the color factor $\xi^a \xi^a = 16/3$, the expression for the amplitude J in eq.(4) can be greatly simplified:

$$J \approx \frac{32}{3} \frac{I}{\Delta} \tag{5}$$

with I given by

$$I = \langle R_{1S} | \, r \, | R_{2P} \rangle \,. \tag{6}$$

One can notice that the overlap integral I describes the matrix element for the electric dipole transition[3] $\chi_{b0}(2P) \to \gamma \, \Upsilon$, and that the rate of the latter transition is given by the well known formula

$$\Gamma(\chi_{b0}(2P) \to \gamma \, \Upsilon) = \frac{4}{9} \, \alpha \, Q_b^2 \, \omega_\gamma^3 \, |I|^2 \,, \tag{7}$$

where $Q_b = -1/3$ is the electric charge of the b quark and $\omega_\gamma \approx 0.74$ GeV is the energy of the emitted photon.

Using the equations (3) and (5) one can thus relate the rates of the two transitions as

$$\frac{\Gamma(\chi_{b0}(2P) \to \eta \, \eta_b)}{\Gamma(\chi_{b0}(2P) \to \gamma \, \Upsilon)} \approx \frac{\pi^3}{3 \, \alpha} \frac{p_\eta \, f_\eta^2 \, m_\eta^4}{\omega_\gamma^3 \, m_b^2 \, \Delta^2} \approx 0.2 \left(\frac{f_\eta}{0.16 \text{ GeV}} \right)^2 \left(\frac{1 \text{ GeV}}{\Delta} \right)^2 \tag{8}$$

---

[3]The form factor effects are neglected here for both the hadronic and the radiative transitions. The corrections due to those effects are smaller than the main uncertainty in the discussed calculation related to the Green function $\mathcal{G}$.

with p_η being the momentum of the emitted η meson. According to PDG[4] the branching ratio for the radiative decay is $B(\chi_{b0}(2P) \to \gamma \Upsilon) = 0.9 \pm 0.6\%$. If the central value of the current data can serve as a reasonable guideline, the branching ratio of the discussed transition $\chi_{b0}(2P) \to \eta \, \eta_b$ can thus be estimated as likely exceeding 10^{-3}, at which level the transition is hopefully within the reach of the experiment.

References

[1] M.B. Voloshin, Yad.Fiz. **35**, 1016 (1982); Sov.J.Nucl.Phys. **35**, 592(1982).

[2] B.A. Kniehl *et.al.*, Phys.Rev.Lett. **92**, 242001 (2004).

[3] L. Marcantonio *et.al.*, UKQCD Coll., Nucl.Phys.Proc.Suppl. **94**, 363(2001).

[4] S. Eidelman *et.al.*, Particle Data Group, Phys.Lett.B **592**, 1 (2004).

[5] S. Blusk (for CLEO Coll.), ArXive:hep-ex/0410048, Oct. 2004.

[6] D.J. Gross, S.B. Treiman, and F. Wilczek, Phys.Rev. Phys.Rev.D **19**, 2188 (1979); V.A. Novikov *et.al.*, Nucl.Phys.B **165**, 55 (1980).

[7] K. Gottfried, Phys.Rev.Lett. **40**, 598 (1978).

[8] M.B. Voloshin, Nucl.Phys.B **154**, 365 (1979).

[9] M. Voloshin and V. Zakharov, Phys.Rev.Lett. **45**, 80 (1980).

[10] M.B. Voloshin, Phys.Lett.B **562**, 68 (2003).